Missouri University of Science & Technology — Department of Computer Science
Fall 2022 — **CS 5408: Game Theory for Computing**

Homework 6: Advanced Solution Concepts

Instructor: *Sid Nadendla* — **Due:** *Dec 9, 2022*

Problem 1 Correlated Equilibrium *5 pts.*

Every day, Alice drives from her house in the north to her job in the south along route 66. Bob drives from his house in the west to his job in the east along route 99. The two roads intersect in the middle of nowhere, where there are currently no stop-signs or traffic-lights. Alice and Bob like to get to work without delays, but they are averse to crashing. This means that they prefer to drive through the intersection without stopping if they can get through safely. Stopping and then proceeding safely through the intersection is preferred by both to crashing. The situation is summarized in the normal-form game below, where Alice is the row-player and Bob is the column-player.

	Go	Stop
Go	-100, -100	10, 5
Stop	5, 10	5, 5

(a) Find all pure-strategy and mixed-strategy Nash equilibria of the game.

(b) Show how the introduction of a traffic light can be modeled as a correlated strategy.

(c) Design and show a contract based on traffic lights, whose correlated equilibrium improves the social welfare (sum of utilities of all players) of the game.

Problem 2 Perfect Equilibrium *5 pts.*

Consider the following game:

	L	R
U	2, 0	1, 1
D	1, 1	1, 0

(a) Prove that, the strategy profile where the row player choosing U with any probability $p \in [\frac{1}{2}, 1)$, and the column player always choosing R, is a mixed strategy NE.

(b) Show that none of these mixed strategy equilibria are trembling-hand perfect.

Problem 3 Evolutionary Game Theory ***5 pts.***

Rock-Paper-Scissors is a game in which Rock (R) beats Scissors (S), Scissors beats Paper (P) and Paper beats Rock, so that the relation 'X beats Y' is *intransitive*. Such intransitive relationships are observed when species interact with each other with individual power/fitness levels. Examples include

- *E. Coli* (a microbe in our guts),
- *Uta Stansburiana* (a California lizard and the first species to which RPS was successfully applied),
- *Ischnura Elegans* (a damselfly), and,
- *Paracercis Sculpta* (a marine isopod).

The interactions amongst the above species can be modeled using a modified Rock-Paper-Scissors game, which is given below.

	Rock	Paper	Scissors
Rock	α, α	-1, 1	1, -1
Paper	1, -1	α, α	-1, 1
Scissors	-1, 1	1, -1	α, α

Prove that the symmetric Nash equilibrium $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is not an evolutionary stable strategy for any $\alpha > 0$.